A non-Lorentz transformation as an alternative to the rainbow model

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Abstract

In Lorentz violation models, the rainbow model is usually discussed, as the rainbow model can make the energy of a particle have a limit rather than be infinite derived from the Lorentz transformation, which is considered to be necessary in the theory of Quantum Gravity. However, this paper shows that it isn't necessary to restrict the speed of light to be a constant when we just agree with the next three principles: (1)we can define the time in the whole space with a prescribed clock synchronization, (2)the time-space is uniform and the space is isotropic and (3)all the inertial systems are equivalent. And base on the above three principles and as a result of variable speed of light, we can construct a general coordinate transformation to satisfy the symmetry of inertial systems and construct a non-Lorentz transformation between inertial systems to make the energy of a particle have a limit, which is the same as the rainbow model.

In addition, in recent papers, as a test at the ultrahigh energy scale, the rainbow model is used to study the Gamma ray burst, such as the GRB 160509A event, which was strongly suggested that there exists a linear relation between the variable speed of light and the photon's energy. So we also analyzed the same event and we found that our model also support the same conclusion as the rainbow model and there was some correlation between our model and the rainbow model. In final we briefly discussed how to verify the two models in the future particle's experiments at the ultrahigh energy scale.

Keywords

Lorentz transformation; Gamma ray burst; rainbow model; variable speed of light; time-space scaling limit; time lag

1. Introduction

The Lorentz invariance violation (LIV) have been arised in various frameworks and theories of Quantum Gravity, such as the string theory [1-3], loop quantum theory [4,5], and non-commutative times-space theory [6]. The most common model used for Lorentz violation is the rainbow model proposed in the double special relativity (DSR), which introduces the Planck length as another constant between inertial systems, thus modifying the particle's energy-momentum dispersion relation at the

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Planck energy scale. The usual form of the rainbow model corresponding to the particle is as follows (taking c=1) [7,8]

$$[1+\chi_1(\frac{E}{E_{LV}})^1+\chi_2(\frac{E}{E_{LV}})^2+O(\frac{E}{E_{LV}})^3]E^2-\boldsymbol{p}^2=m_0^2 \tag{1}$$

where E is the total energy , m_0 is the rest mass, $\boldsymbol{p}$ is the momentum, and E_{LV} is the energy scale at which Lorentz violating effects become strong, the couplings $\chi_s=\pm1$ (s=1, 2) are determined by the dynamical framework being studied.

The introduction of the double special relativity provides a new perspective for the development of quantum gravity. For example, the application of Eq. (1) can avoid the divergence of the black hole's temperature in Liu's work [9,10] and solve the singularity problem in the Big Bang model in Ling's work [11,12]. And Lots of researches have been done to verify the correction terms in Eq. (1), such as Bolmont [13] used the HETE-2 gamma ray burst data to constrain the energy scale E_{LV} to be at least 2×10^{15} GeV at 95% Confidence Level, while Nilsson [14] applied the Hubble+SNIa+BAO (BOSS+Lyman α)+CMB data to constrain E_{LV} to be at least the order of 10^{16} GeV at 1σ, or even 10^{17} GeV at 3σ. Additionally, based on the first-order approximation of Eq. (1), Xu [15,16] analyzed the GRB 160509A event and pointed out that there exists a linear relation between the variable speed of light and the photon's energy, and $E_{LV}=3.6\times10^{17}$ GeV was obtained.

Although the rainbow model made progress in some researches, more work needs to be done to verify the model in future particle's experiments at the ultrahigh energy scale. Importantly, the rainbow model is somewhat complicated, which leads the correction terms of the parameter χ_s are unclear physically.

As the rainbow model provides a possibility of variable speed of light, this paper re-investigated the relationship between the symmetry of inertial systems and the Lorentz transformation. We found that the Lorentz transformation was not a necessary condition for satisfying the symmetry of inertial systems. And therefore we construct a general function of the variable speed of light to make the inertial systems to be equivalent. Then we will focus on the comparison of the modified energy-momentum dispersion relation proposed in our model and the rainbow model, especially the application of the energy-momentum dispersion relation corresponding to the rainbow model in the work of Xu's work [16]. At last we will discuss how to verify the two models in the future particle's experiments at the ultrahigh energy scale.

2. Possibility of variable speed of light

As we know the rainbow model presents that the speed of light maybe associated with its energy [15,16], then here we proposed a general hypothesis that: For a light source in vacuum, when it moves at a velocity $\boldsymbol{v}$ relative to an observer in vacuum, then the observed (by the observer) speed of light emitted by the light source is $\boldsymbol{nc}$, where n is a dimensionless quantity, $\boldsymbol{c}$ is the speed of light in vacuum. Obviously, in order not to violate some fundamental principles and experiments, we should imposed some rules on the parameter n as follows

1. As stated in the special relativity, firstly, it should be possible to define the time in the whole space with a prescribed clock synchronization, that is, for a specific inertial system, we can calibrate the clock in the inertial system to synchronize by the speed of light emitted by a light source that is stationary in the specific inertial system.

So this principle requires that

$$n(v=0,\boldsymbol{c})=1 \tag{2}$$

2. As a general concept of time and space, that the time-space is uniform and the space is isotropic. So this principle requires that

$$n(v,c)=n(-v,c)=n(v,-c)=n(-v,-c) \tag{3}$$

3. In addition, we should agree with that all the inertial systems are equivalent.

Based on the above assumption, now we discuss the coordinate transformation between the two inertial systems $S(\boldsymbol{x},\boldsymbol{y},z,t)$ and $S'(\boldsymbol{x}',\boldsymbol{y}',z',t')$, which move at a velocity $\boldsymbol{v}$ relative to each other.

Firstly, for simplicity, we assume the three spatial coordinates of the two coordinate systems are parallel to each other, and the direction of $\boldsymbol{v}$ along $\boldsymbol{x}$-axis or $\boldsymbol{x}'$-axis, then there is $\boldsymbol{y}=\boldsymbol{y}'$, $z=z'$.

Secondly, since the time-space is uniform, the coordinate transformation between S and S' should be in a linear form, then we assume that

$$\boldsymbol{x}=\gamma(\boldsymbol{x}'+\boldsymbol{v}t') \tag{4}$$

Where $\gamma=\gamma(\boldsymbol{v},\boldsymbol{c})$ is a proportionality constant.

Similarly, because of the symmetry of S and S', it has

$$\boldsymbol{x}'=\gamma'[\boldsymbol{x}+(-\boldsymbol{v})t] \tag{5}$$

Where $\gamma'=\gamma'(-\boldsymbol{v},-\boldsymbol{c})$ is a proportionality constant.

As the S and S' are equivalent, then it means

$$\gamma(\boldsymbol{v},\boldsymbol{c})=\gamma'(-\boldsymbol{v},-\boldsymbol{c}) \tag{6}$$

Now we will solve γ. If the light signal is emitted by the light source at the moment that the origin of S and S' are coincides, then based on the above assumption on the speed of light, we will obtain

$$\begin{cases} \boldsymbol{x}^2+\boldsymbol{y}^2+\boldsymbol{z}^2=(\boldsymbol{c}t)^2 \\ \boldsymbol{x}'^2+\boldsymbol{y}'^2+\boldsymbol{z}'^2=(\boldsymbol{n}\boldsymbol{c}t')^2 \\ \boldsymbol{y}=\boldsymbol{y}'=0 \\ \boldsymbol{z}=\boldsymbol{z}'=0 \end{cases} \quad \text{and} \quad \begin{cases} \boldsymbol{x}'^2+\boldsymbol{y}'^2+\boldsymbol{z}'^2=(\boldsymbol{c}t')^2 \\ \boldsymbol{x}^2+\boldsymbol{y}^2+\boldsymbol{z}^2=(\boldsymbol{n}\boldsymbol{c}t)^2 \\ \boldsymbol{y}=\boldsymbol{y}'=0 \\ \boldsymbol{z}=\boldsymbol{z}'=0 \end{cases} \tag{7}$$

The first formula of Eq. (7) represents that the light source is stationary in S, then the observed speed of light by an observer in S is $\boldsymbol{c}$, while the observed speed of light by another observer in S' is nc.

Similarly, because of the symmetry of S and S', when the light source is stationary in S', then the observed speed of light by an observer in S' is $\boldsymbol{c}$, while the observed speed of light by another observer in S is $\boldsymbol{nc}$, which corresponds to the second formula of Eq. (7).

From Eq. (4)~Eq. (7), we can solve γ and obtain the coordinate transformation between S and S'

$$\begin{cases} \boldsymbol{x}' = \gamma(\boldsymbol{x} - \boldsymbol{v}t) \\ t' = \gamma(t - \dfrac{\boldsymbol{v}}{k^2}\boldsymbol{x}) \end{cases} \tag{8}$$

where $\gamma(\boldsymbol{v},\boldsymbol{c}) = 1/\sqrt{1-\boldsymbol{v}^2/k^2}$, $k(\boldsymbol{v},\boldsymbol{c}) = \sqrt{n\boldsymbol{v}\boldsymbol{c}^2/(n\boldsymbol{c}-\boldsymbol{c}+\boldsymbol{v})}$.

Based on Eq. (8) we will obtain that

$$\frac{d\boldsymbol{x}'}{dt'} = \frac{d\boldsymbol{x} - \boldsymbol{v}dt}{dt - \dfrac{\boldsymbol{v}}{k^2}d\boldsymbol{x}} = \frac{d\boldsymbol{x}/dt - \boldsymbol{v}}{1 - \dfrac{\boldsymbol{v}}{k^2(\boldsymbol{v},\boldsymbol{c})}\dfrac{d\boldsymbol{x}}{dt}} = f(\boldsymbol{v},\boldsymbol{c}) \tag{9}$$

As we know the direction of vector $\boldsymbol{v}$ and $\boldsymbol{c}$ maybe along the positive $\boldsymbol{x}$-axis (or $\boldsymbol{x}'$-axis) or along the negative $\boldsymbol{x}$-axis (or $\boldsymbol{x}'$-axis), and if we distinguish the direction of $\boldsymbol{v}$ and $\boldsymbol{c}$ by the positive and negative signs, then we will obtain four different combinations, that is, (v, c), (v, -c), (-v, c), (-v, -c), which represents four cases corresponding to the different direction of $\boldsymbol{v}$ and $\boldsymbol{c}$.

Based on Eq. (9), we can obtain

$$\begin{cases} \text{when } \dfrac{dx}{dt} = c, \quad \dfrac{dx'}{dt'} = f(v,c) = \dfrac{c-v}{1-\dfrac{v}{k^2(v,c)}c} = f(-v,c) = \dfrac{c+v}{1+\dfrac{v}{k^2(-v,c)}c} = nc \\ \text{when } \dfrac{dx}{dt} = -c, \quad \dfrac{dx'}{dt'} = f(v,-c) = \dfrac{(-c)-v}{1-\dfrac{v}{k^2(v,-c)}(-c)} = f(-v,-c) = \dfrac{-c+v}{1+\dfrac{v}{k^2(-v,-c)}(-c)} = -nc \\ \text{when } \dfrac{dx'}{dt'} = c, \quad \dfrac{dx}{dt} = f(-v,c) = \dfrac{c+v}{1+\dfrac{v}{k^2(-v,c)}c} = f(v,c) = \dfrac{c-v}{1-\dfrac{v}{k^2(v,c)}c} = nc \\ \text{when } \dfrac{dx'}{dt'} = -c, \quad \dfrac{dx}{dt} = f(-v,-c) = \dfrac{-c+v}{1+\dfrac{v}{k^2(-v,-c)}(-c)} = f(v,-c) = \dfrac{(-c)-v}{1-\dfrac{v}{k^2(v,-c)}(-c)} = -nc \end{cases} \tag{10}$$

Eq. (10) can also be expressed in a vector form as

$$\begin{cases} \text{when } \dfrac{d\boldsymbol{x}}{dt} = \boldsymbol{c}, \quad \dfrac{d\boldsymbol{x}'}{dt'} = f(\boldsymbol{v},\boldsymbol{c}) = n\boldsymbol{c} \\ \text{when } \dfrac{d\boldsymbol{x}'}{dt'} = \boldsymbol{c}, \quad \dfrac{d\boldsymbol{x}}{dt} = f(\boldsymbol{v},\boldsymbol{c}) = n\boldsymbol{c} \end{cases} \tag{11}$$

Eq. (10) and Eq. (11) implies that Eq. (8) is the solution of Eq. (7) in turn. In addition, it implies that the S and S' are equivalent, that is, no matter the light source is stationary in S or S', the observed speed of light is $\boldsymbol{c}$ by the observer who is stationary relative to the light source, while by another observer who is moving at a velocity $\boldsymbol{v}$ relative to the light source, the observed speed of light is $n\boldsymbol{c}$.

As stated above, based on the different signs of $\boldsymbol{v}$ and $\boldsymbol{c}$, one can obtain four

combination of k, that is, $k(v,c)$, $k(-v,c)$, $k(v,-c)$, $k(-v,-c)$. However, it is easy to prove that $k(v,c)=k(-v,-c)$, $k(-v,c)=k(v,-c)$, which is the result of a symmetric transformation in Eq. (6). But from Eq. (4)~Eq. (7), it doesn't require that $k(v,c)$ is equal to $k(-v,c)$. In fact $k(v,c)$ and $k(-v,c)$ represent two different cases or two independent events, they cannot be linked together by the symmetry in one event.

The two independent events or cases corresponding to the same or different signs of $\boldsymbol{v}$ and $\boldsymbol{c}$ in formula k leads to the two solutions of Eq. (7). But for our world, the coordinate transformation between inertial systems must be unique, so we must discard one of the two solutions. Or in other words, for example, Eq. (10) shows that, when $\mathrm{d}x/\mathrm{d}t=c$, then $\mathrm{d}x'/\mathrm{d}t=f(v,c)=f(-v,c)=nc$, where $f(v,c)$ and $f(-v,c)$ represent two different cases, which corresponds to the two solutions of Eq. (7) or Eq. (8). So we must take one of the two solutions as a description of our world.

From some basic principles or empirical facts, the following we will take the solution that the sign of $\boldsymbol{v}$ and $\boldsymbol{c}$ is the same in formula k as the unique transformation between S and S', and the reason why we choose the solution will be further expressed in section 3.

Importantly, the forms of Eq. (8) are similar to the Lorentz transformation, and it is easy to prove that the Maxwell's Equations are covariant based on Eq. (8).

Here it should be noted that, we just try to allow the speed of light to be possible to change, like the rainbow model, and on the basis we try to give some qualifications on the parameter n to make the time-space meet the above three principles, then we obtain a self-consistent coordinate transformation equation between inertial systems. The idea is different from the model of double special relativity, which introduce a second constant besides the speed of light, and also different from the model of Standard Model Extension (SME), which the observer Lorentz transformation is reserved while the particle Lorentz transformation is violated by introducing extra fields. So far we didn't introduce any fields or new constants, and it seems that the property of time-space are complicated. In fact we can derive the time-space metric base on Eq. (8)

$$ds^2 = -k^2dt^2 + d\boldsymbol{x}^2 \tag{12}$$

In addition, based on Eq. (8), we can obtain the particle's energy-momentum dispersion relation as

$$E^2 = \boldsymbol{p}^2k^2 + E_0^2 \tag{13}$$

Where $E_0=m_0k^2$ represents the particle's rest energy with rest mass m_0, $E=\gamma m_0k^2$ represents the particle's total energy, $\boldsymbol{p}=\gamma m_0\boldsymbol{v}$ represents the particle's momentum.

3. Limit of a particle's energy

As we know, in Lorentz transformation the particle's energy tends to be infinite when its velocity is close to the speed of light, however, the DSR model introduces a new constant as the limit energy of all the particles, which is considered to be necessary in the theory of Quantum Gravity. As stated above that the rainbow model involves some parameters that are not yet identified physically. However, inspired by

the idea of rainbow model, we found that Eq. (8) can derive the same result as the rainbow model.

From the taken solution of Eq. (8), the time-space scaling factor is

$$\gamma = \frac{1}{\sqrt{1 - v^2 / k^2}} = \frac{1}{\sqrt{\frac{1 - v / c}{n}(n + \frac{v}{c})}} \tag{14}$$

Note that in Eq. (14), because the sign of ***v*** and ***c*** is the same in our taken solution, for simplicity, we take the signs of ***v*** and ***c*** is positive, which will not affect the sign of *v*/*c*.

Eq. (14) inspires us that when v=c, it is possible that γ does not tend to be infinite if we assume n=0 at the same time. So we can construct an expression for *n*. As *n* has been constrained in Eq. (2) and Eq. (3), we try to take the following expression for *n*

$$n = \frac{1}{1 - Q}(1 - Q^{1 - v^2 / c^2}) \tag{15}$$

where Q is a constant determined by the experiments or other theories.

Figure 1 shows the *n*~*v* curve when taking $Q = (1/2)^{10^6}$ as an example.

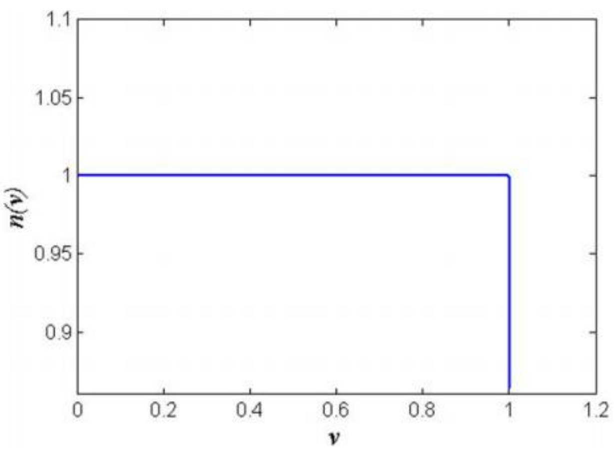

Fig. 1 *n*(*v*)~*v* curve (Taking *c*=1)

Thus, the limit of time-space scaling factor γ and particle's total energy E are respectively

$$\lim_{v \to c} \gamma = \lim_{v \to c} \frac{1}{\sqrt{1 - v^2 / k^2}} = \lim_{v \to c} \frac{1}{\sqrt{(\frac{1 - v / c}{n})(n + \frac{v}{c})}} = \sqrt{\frac{2 \ln Q}{Q - 1}} \tag{16}$$

$$\lim_{v \to c} E = \lim_{v \to c} \gamma m_0 k^2 = E_{QG} = \frac{m_0 c^2}{[1 - 0.5(Q - 1) / \ln Q]} \sqrt{\frac{2 \ln Q}{Q - 1}} \tag{17}$$

4. Modified arrival time lag effect

It can be seen from Fig.1 that the modified particle's energy-momentum dispersion relation will be back to the Lorentz case at low or medium energy. Then we discuss the particle at the ultrahigh energy scale.

When $v \sim c$ for an ultra-relativistic particle, it can be obtained from Eq. (15) that (taking c=1)

$$n = \frac{1}{1-Q}(1-Q^{1-v^2}) = \frac{1}{1-Q}[1-Q^{(1+v)(1-v)}] \approx \frac{1}{1-Q}[1-Q^{2(1-v)}] \approx \frac{2\ln Q}{Q-1}(1-v) \tag{18}$$

Then

$$\begin{aligned}
\frac{E}{E_{QG}} &= \frac{m_0 k^2}{\sqrt{1-v^2/k^2}} / [\frac{m_0 c^2}{[1-0.5(Q-1)/\ln Q]}\sqrt{\frac{2\ln Q}{Q-1}}] \\
&= \frac{[1-0.5(Q-1)/\ln Q]}{\sqrt{2\ln Q/(Q-1)}} \frac{nv/(n-1+v)}{\sqrt{(1-v)(1+v/n)}} \\
&\approx \frac{[1-0.5(Q-1)/\ln Q]}{\sqrt{2\ln Q/(Q-1)}} \frac{2\ln Q/(Q-1)v}{2\ln Q/(Q-1)-1} \frac{1}{\sqrt{1-v+v(Q-1)/(2\ln Q)}} \\
&\approx \frac{[1-0.5(Q-1)/\ln Q]}{\sqrt{2\ln Q/(Q-1)}} \frac{2\ln Q/(Q-1)}{2\ln Q/(Q-1)-1} \frac{1}{\sqrt{(Q-1)/(2\ln Q)+(1-v)}} \\
&\approx \frac{1}{\sqrt{2\ln Q/(Q-1)}}[\sqrt{\frac{2\ln Q}{Q-1}} - \frac{1}{2}(\frac{2\ln Q}{Q-1})^{3/2}(1-v)] \\
&= 1 - \frac{\ln Q}{Q-1}(1-v)
\end{aligned} \tag{19}$$

From Eq. (19) we can obtain that

$$\frac{v}{c} = 1 - \frac{Q-1}{\ln Q} + \frac{Q-1}{\ln Q}\frac{E}{E_{QG}} \tag{20}$$

Eq. (20) implies that for an ultra-relativistic particle described by Eq. (13) and Eq. (15), its velocity is proportional to its energy.

Multiplying mc^2 on both sides of Eq. (20), we can obtain the equation for the photons

$$pc = E(1 - \frac{Q-1}{\ln Q} + \frac{Q-1}{\ln Q}\frac{E}{E_{QG}}) \tag{21}$$

where p is the photon's momentum , and E is the photon's energy.

If $E/E_{QG} \sim 0$, then based on Eq. (21) and Ref. [17], we can obtain that

$$\frac{\partial E}{\partial p} = \frac{1}{1+\frac{Q-1}{\ln Q}\frac{E}{E_{QG}}} \approx 1 - \frac{Q-1}{\ln Q}\frac{E}{E_{QG}} \tag{22}$$

Further based on Ref. [17], we can obtain the comoving distance traversed by a photon, emitted at redshift z and traveling up to redshift 0

$$x(z,E) = \frac{c}{H_0}\int_0^z [1 - \frac{Q-1}{\ln Q}(\frac{E^0}{E_{QG}})(1+z')] \frac{dz'}{\sqrt{\Omega_m(1+z')^3 + \Omega_\Lambda}} \tag{23}$$

where E^0 is the photon's energy measured at present, Ω_m, Ω_Λ and H_0 are the cosmological parameters evaluated today.

Then based on Eq. (23) we can obtain the arrival time lag due to the modified

energy-momentum dispersion relation in Eq. (13) as

$$\Delta t=\frac{Q-1}{\ln Q}\frac{1}{H_0}\frac{E^0}{E_{QG}}\int_0^z\frac{(1+z')dz'}{\sqrt{\Omega_m(1+z')^3+\Omega_\Lambda}}\propto E^0 \tag{24}$$

In Ref. [16], the first-order approximation of Eq. (1) (that is, χ_1=1, χ_2=0) is applied to analyze the photon, and the arrival time lag was obtained as

$$\Delta t'=\frac{1}{H_0}\frac{E^0}{E_{LV}}\int_0^z\frac{(1+z')dz'}{\sqrt{\Omega_m(1+z')^3+\Omega_\Lambda}}\propto E^0 \tag{25}$$

By analyzing the photon's arrival time lag with different energies in the GRB 160509A event based on Eq. (25), the Ref. [16] concluded that there exists a linear relation between the observed speed of light and the photon's energy in cosmological space as

$$\frac{c_1}{c}=1-\frac{E}{E_{LV}} \tag{26}$$

where c_1 is the observed speed of light, E_{LV}=3.6×10^{17} GeV.

It is worth noting that the data and conclusion in Ref. [16] can be applied equally to an ultra-relativistic massive particle [17], such as the energetic neutrino.

Comparing Eq. (24) with Eq. (25), it can be seen that the modified energy-momentum dispersion relation in Eq. (13) is also consistent with the conclusion in Ref. [16].

More importantly, as many experiments have restricted the violation of Lorentz transformation, such as the experiments in Ref. [18-26], it means the value of Q is very small (when Q=0 it means $n\equiv 1$), and in our previous work [27], we have restricted that $Q<(1/e)^{10^6}$, which the corresponding time-space scaling factor γ limit is 1414.2, while based on the conclusion in Ref. [15-17], we can obtain that $E_{QG}\approx$-1/(lnQ)E_{LV}, which means the restrictions on E_{LV} in previous work can also be applied on E_{QG}.

4. Conclusion

In this paper we have discussed the relationship between the speed of light and the symmetry of inertial systems, we found that it was not necessary to restrict the speed of light to be a constant when we just agree with that: (1)we can define the time in the whole space with a prescribed clock synchronization, (2)the time-space is uniform and the space is isotropic and (3)all the inertial systems are equivalent. Therefor we construct a general coordinate transformation between inertial systems to satisfy the symmetry of inertial systems. And we also construct a non-Lorentz transformation between inertial systems, which same as the rainbow model, it can make the particle's energy have a limit.

By comparing with the rainbow model applied in the ultrahigh energy scale, we found that the modified energy-momentum dispersion relation in our model can also support the conclusion in Ref. [16]. And the particle's energy limit in our model is somewhat associated with which in the rainbow model. However, the data in Ref. [15,16] still cannot verify Eq. (1) or Eq. (13), as the value of Q should be determined in the massive particle's experiments, which is expected in the future's energetic

neutrino experiments. In the energetic neutrino experiments, as Eq. (20) shown, a neutrino described by Eq. (1) and Eq. (13) will show different behaviors.

However, compared with the rainbow model, Eq. (13) has clear correction terms in physics, and the model has just one undetermined parameter (the value of Q), which depends on the limit of time-space scaling factor γ. In addition, same as the rainbow model, if the value of Q is not equal to 0, then it will affect the current black hole model, which we will discuss it in the next paper.

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